Field Expansions in Cavities Containing Gyrotropic Media*

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Summary—**The electric and magnetic eigenvectors for cavities containing media with tensor characteristics are examined, and explicit formulas for the electric and magnetic fields are obtained in terms of volume and boundary sources.**

I. Introduction

Consider a cavity enclosed by a perfectly conducting wall provided with an opening S'. The cavity is excited by electric and magnetic volume currents $\bar{J}$ and $\bar{J}_m$, and through the opening in the wall, where the tangential component of $\bar{E}$ is given. We address ourselves to the task of determining the interior fields when the cavity contains an inhomogeneous anisotropic medium. This problem has been solved a long time ago for a cavity containing a homogeneous isotropic medium.[1] We want to extend these results to our new configuration, where, for example, the medium inside could typically be a ferrite in a dc magnetic field. The successive steps are not very novel, but it was considered worthwhile to spell them out carefully for reference purposes. Our analysis, which relies on previously published material,[2–6] will limit itself to media in which the constitutive tensors ϵ and μ have the following properties:

1) They are hermitian (*i.e.*, $\epsilon = [\tilde{\epsilon}]^*$ and $\mu = [\tilde{\mu}]^*$).
2) They yield positive-definite quadratic forms $A^* \cdot \epsilon \cdot A$ and $A^* \cdot \mu \cdot A$.[7]
3) They are frequency independent.

A large number of materials satisfy these requirements over a reasonable frequency range. Examples are: a) ordinary dielectrics (homogeneous or inhomogeneous), b) certain plasmas and c) ferrites in weak dc magnetic fields, and far from gyromagnetic resonance. To solve the expansion problem, we must introduce two kinds of eigenvectors, electric and magnetic.

II. Electric Eigenvectors

The electric eigenvectors are sourceless sinusoidal fields which can be sustained by a cavity completely surrounded by metal. They will, consequently, satisfy the *electric* conditions on the metallic boundary S:

$$\bar{n} \times \bar{E} = 0$$
$$\text{div}\,(\epsilon \cdot \bar{E}) = 0.$$

We want to examine the completeness properties of these eigenvectors. Following the steps outlined in Slater,[1] we will consider a linear transformation whose operator generalizes the vector Laplacian. This operator is

$$\mathcal{L}\bar{f} = \text{grad div}\,(\epsilon \cdot \bar{f}) - \epsilon^{-1} \cdot \text{curl}(\mu^{-1} \cdot \text{curl}\,\bar{f}) \tag{1}$$

and the class of vectors $\bar{f}$ we consider are those which have the necessary derivatives and satisfy the *electric* boundary conditions. To establish completeness, we shall follow a pattern used over and over again in similar problems. We first define a scalar product,

$$\langle \bar{u}, \bar{v} \rangle = \iiint_V \bar{u}^* \cdot \epsilon \cdot \bar{v}\, dV. \tag{2}$$

This type of scalar product, with the restrictions imposed on ϵ, satisfies the properties of the scalar product in a Hilbert space.[8] With respect to that scalar product, the transformation is self-adjoint and negative-definite. Indeed:

$$1)\ \langle \mathcal{L}\bar{u}, \bar{v} \rangle = \iiint_V [\text{grad div}\,(\epsilon^* \cdot \bar{u}^*) - (\epsilon^{-1})^* \cdot \text{curl}\,[(\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*] \cdot \epsilon \cdot \bar{v}\, dV.$$

By using elementary formulas of vector analysis, this expression can be transformed into

$$\iiint \text{grad div}\,(\epsilon^* \cdot \bar{u}^*) \cdot \epsilon \cdot \bar{v}\, dV = \iint_S \bar{n} \cdot [\text{div}\,(\epsilon^* \bar{u}^*) \epsilon \cdot \bar{v}] dS - \iiint_V \text{div}\,(\epsilon^* \cdot \bar{u}^*)\, \text{div}\,(\epsilon \cdot \bar{v}) dV,$$

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[7] This entails that the stored frequency energy has a positive average value. Notice that ϵ and μ are the relative, dimensionless constitutive parameters of the medium.

[8] See, *e.g.*, B. Friedman, "Principles and Techniques of Applied Mathematics," John Wiley and Sons, Inc., New York, N. Y.; 1956.

and

$$-\iiint_V \{(\epsilon^{-1})^* \cdot \text{curl}\,[(\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*]\} \cdot \epsilon \cdot \bar{v} dV$$
$$= -\iiint_V \{\text{curl}\,[(\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*] \cdot (\tilde{\epsilon}^{-1})^*\} \cdot \epsilon \cdot \bar{v} dV$$
$$= -\iiint_V \text{curl}\,[(\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*] \cdot \bar{v} dV$$
$$= -\iint_S \{[(\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*] x \bar{v}\} \cdot \bar{n} dS$$
$$-\iiint_V \text{curl}\,\bar{v} \cdot (\mu^{-1})^* \cdot \text{curl}\,\bar{u}^* dV.$$

We have used the property that $(\tilde{\epsilon})^* = \epsilon$ and $(\bar{a} \cdot \mathcal{A}) \cdot (\mathcal{B} \cdot \bar{b}) = \bar{a} \cdot (\mathcal{A} \cdot \mathcal{B}) \cdot \bar{b}$, where $\bar{a}\bar{b}$ are vectors, and $\mathcal{A}$, $\mathcal{B}$ are tensors. For the range of vectors considered in the transformation, both surface integrals vanish, and we find

$$\langle \mathcal{L}\bar{u}, \bar{v} \rangle = -\iiint_V [\text{div}\,\epsilon \cdot \bar{u}]^* \,\text{div}\,\epsilon \cdot \bar{v} dV$$
$$-\iiint_V \text{curl}\,\bar{v} \cdot (\mu^{-1})^* \cdot \text{curl}\,\bar{u}^* dV. \quad (3)$$

The calculation of $\langle \bar{u}, \mathcal{L}\bar{v} \rangle$ proceeds along very similar lines. It yields

$$\langle \bar{u}, \mathcal{L}\bar{v} \rangle = -\iiint_V \text{div}\,(\epsilon \cdot \bar{v})\,\text{div}\,(\bar{u}^* \cdot \epsilon) dV$$
$$-\iiint_V \text{curl}\,\bar{u}^* \cdot \mu^{-1} \cdot \text{curl}\,\bar{v} dV. \quad (4)$$

Expressions (3) and (4) are equal because

$$\bar{u}^* \cdot \epsilon = \tilde{\epsilon} \cdot \bar{u}^* = \epsilon^* \cdot \bar{u}^*$$

and

$$\text{curl}\,\bar{v} \cdot (\mu^{-1})^* \cdot \text{curl}\,\bar{u}^*$$
$$= \text{curl}\,\bar{u}^* \cdot (\tilde{\mu}^{-1}) \cdot \text{curl}\,\bar{v} = \text{curl}\,\bar{u}^* \cdot \mu^{-1} \cdot \text{curl}\,\bar{v}.$$

Thus we conclude that the transformation is self-adjoint.

2) $\langle \mathcal{L}\bar{u}, \bar{u} \rangle$ can be found from (3) and (4) to be

$$\langle \mathcal{L}\bar{u}, \bar{u} \rangle = -\iiint_V |\,\text{div}\,\epsilon \cdot \bar{u}\,|^2 dV$$
$$-\iiint_V \text{curl}\,\bar{u}^* \cdot \mu^{-1} \cdot \text{curl}\,\bar{u} dV.$$

This form is always negative for $\bar{u}$ nonzero. Setting $\langle \mathcal{L}\bar{u}, \bar{u} \rangle$ equal to zero yields

$$\text{div}\,\epsilon \cdot \bar{f}_0 = 0$$
$$\text{curl}\,\bar{f}_0 = 0,$$

where $\bar{f}_0$ is perpendicular to the boundary. Clearly, $\bar{f}_0$ is the electrostatic field which arises when the boundaries are metallized and set at a constant potential, and the medium inside the cavity has a (complex) dielectric constant ϵ, and contains no charges. This field is zero when the volume is singly-bounded. The proof of this statement is as follows. First notice that $\bar{f}_0$ is irrotational, and can be written as grad ϕ_0, where

$$\text{div}\,[\epsilon \cdot \text{grad}\,\phi_0] = 0$$
$$\phi_0 = 0 \quad \text{on } S.$$

Applying the divergence theorem, we obtain

$$\iiint_V \text{div}\,[\phi_0^* \epsilon \cdot \text{grad}\,\phi_0] dV$$
$$= \iint_S \phi_0^* [\epsilon \cdot \text{grad}\,\phi_0] \cdot \bar{u}_n dS = 0$$
$$= \iiint_V \phi_0^*\,\text{div}\,[\epsilon \cdot \text{grad}\,\phi_0] dV$$
$$+ \iiint_V \text{grad}\,\phi_0^* \cdot \epsilon \cdot \text{grad}\,\phi_0 dV. \quad (5)$$

The positive definite character of the last integral implies that $\bar{f}_0 = \text{grad}\,\phi_0 = 0$ everywhere. The proof breaks down for a doubly-bounded region, which admits a nonzero solution $\bar{f}_0$. We conclude that our transformation is negative-definite in a singly-bounded volume, but that it will keep this character in a doubly-bounded volume only if we restrict our range to those vectors $\bar{f}$ which do not "contain" any part of $\bar{f}_0$, *i.e.*, have zero projection on the $\bar{f}_0$ in the sense that

$$\iiint_V \bar{f}^* \cdot \epsilon \cdot \bar{f}_0 dV = 0.$$

With these restrictions, we find that the transformation, being self-adjoint and negative-definite, has all the desirable properties of such transformations. In particular, the eigenvectors

$$\text{grad div}\,(\epsilon \cdot \bar{u}_m) - \epsilon^{-1} \cdot \text{curl}\,(\mu^{-1} \cdot \text{curl}\,\bar{u}_m) + k_m^2 \bar{u}_m = 0 \quad (6)$$

form a closed and complete set. They are orthogonal, *i.e.*, the normalized eigenvectors satisfy

$$\iiint_V \bar{u}_m^* \cdot \epsilon \cdot \bar{u}_n dV = 0, \quad (7)$$

where $\bar{u}_m$ and $\bar{u}_n$ correspond to different eigenvalues k_m^2 and k_n^2. The eigenvalues k_n^2 are real and positive, a fact of great physical significance in terms of resonant frequencies. The eigenvectors belong to two classes:

a) Irrotational eigenvectors $\bar{f}_m = \text{grad}\,\phi_m$, where

$$\text{div}\,(\epsilon \cdot \text{grad}\,\phi_m) + \mu_m^2 \phi_m = 0 \qquad \phi_m = 0 \text{ on } S. \quad (8)$$

The functions ϕ_m are orthogonal in the sense that

$$\iiint_V \phi_m^* \phi_n dV = \delta_{mn}. \quad (9)$$

When the region is doubly-bounded, one should not forget to include the eigenvector $\bar{f}_0$ (with eigenvalue zero) in the set of $\bar{f}_m$'s.

b) The eigenvectors of the transformation

$$-\epsilon^{-1}\cdot \text{curl}\,[\mu^{-1}\cdot \text{curl}\,\bar{e}_m] + \nu_m^2 \bar{e}_m = 0$$
$$\bar{n} \times \bar{e}_m = 0 \text{ on } S. \quad (10)$$

It is an easy matter to check that these eigenvectors are orthogonal among themselves, and to the irrotational vectors $\bar{f}_m$.

A complex-square intergrable vector $\bar{a}$ can be expanded in the $\bar{f}_m$ and the $\bar{e}_m$, and the expansion formula takes the form

$$\bar{a} = \sum_m \bar{f}_m \left[\frac{-\iiint_V \phi_m^* \,\text{div}\,(\epsilon\cdot\bar{a}) dV}{\mu_m^2 \iiint_V \phi_m^* \phi_m dV} \right] + \sum_m \bar{e}_m \frac{\iiint_V \text{curl}\,\bar{e}_m^* \cdot \mu^{-1}\cdot \text{curl}\,\bar{a} dV + \iint_S \text{curl}\,\bar{e}_m^* \cdot \mu^{-1}\cdot (\bar{a}\times\bar{n}) dS}{\nu_m^2 \iiint_V \bar{e}_m^* \cdot \epsilon \cdot \bar{e}_m dV}. \quad (11)$$

Clearly the $\bar{e}_m$ form a complete set for the restricted class of vectors for which $\text{div}(\epsilon\cdot\bar{a}) = 0$, and the $\bar{f}_m$ form a complete set for the class of irrotational vectors perpendicular to the boundary. The application of this expansion formula to electric fields will be taken up in Section IV.

III. Magnetic Eigenvectors

The boundary conditions satisfied by the magnetic field at a perfectly conducting wall are of the *magnetic* type

$$\bar{n}\cdot[\mu\cdot\bar{H}] = 0$$
$$\bar{n}\times[\epsilon^{-1}\cdot \text{curl}\,\bar{H}] = 0.$$

They express that the induction $\bar{B}$ is tangent to the walls, and the electric field $\bar{E}$ is perpendicular to the latter. This suggests the kind of linear transformation we should consider. The suitable scalar product is

$$\langle \bar{u}, \bar{v}\rangle = \iiint_V \bar{u}^* \cdot \mu \cdot \bar{v} dV. \quad (12)$$

We follow the steps outlined in Section II, and assert:

1) That the linear transformation

$$M\bar{f} = \text{grad div}\,(\mu\cdot\bar{f}) - \mu^{-1}\cdot \text{curl}\,(\epsilon^{-1}\cdot \text{curl}\bar{f}), \quad (13)$$

where $\mu\cdot\bar{f}$ is tangent to the boundary and $\epsilon^{-1}\cdot \text{curl}\,\bar{f}$ is perpendicular to the boundary, is self-adjoint and negative-definite in a simply-connected region. In a doubly-connected region modifications are caused by the existence of a *magnetostatic field* $\bar{g}_0$ satisfying

$$\text{div}\,(\mu\cdot\bar{g}_0) = 0$$
$$\text{curl}\,\bar{g}_0 = 0$$
$$\mu\cdot\bar{g}_0 \text{ tangent to } S,$$

the modifications being similar to those introduced by the existence of $\bar{f}_0$.

2) That the eigenvectors

$$\text{grad div}\,(\mu\cdot\bar{v}_m) - \mu^{-1}\cdot \text{curl}\,(\epsilon^{-1}\cdot \text{curl}\,\bar{v}_m) + k_m^2 \bar{v}_m = 0 \quad (14)$$

form a closed and complete set, are orthogonal in the sense that the normalized eigenvectors satisfy

$$\iiint_V \bar{v}_m^* \cdot \mu \cdot \bar{v}_n dV = \delta_{mn}, \quad (15)$$

and belong to two classes:

a) Irrotational eigenvectors $\bar{g}_m = \text{grad}\,\psi_m$ where

$$\text{div}\,[\mu\cdot \text{grad}\,\psi_m] + \lambda_m^2 \psi_m = 0$$
$$\bar{n}\cdot\mu\cdot \text{grad}\,\psi_m = 0 \text{ on } S. \quad (16)$$

The ψ_m are orthogonal in the sense of (9), and the $\bar{g}_m$ in the sense of (15).

b) The eigenvectors of the transformation

$$-\mu^{-1}\cdot \text{curl}\,(\epsilon^{-1}\cdot \text{curl}\,\bar{h}_m) + \nu_m^2 \bar{h}_m = 0$$
$$\bar{n}\times(\epsilon^{-1}\cdot \text{curl}\,\bar{h}_m) = 0 \text{ on } S. \quad (17)$$

A few simple steps show that the eigenvalues in (10) and (17) are identical, and that the following connection exists between eigenvectors:

$$\bar{h}_m = \frac{1}{\nu_m} \mu^{-1}\cdot \text{curl}\,\bar{e}_m$$
$$\bar{e}_m = \frac{1}{\nu_m} \epsilon^{-1}\cdot \text{curl}\,\bar{h}_m$$

$$\iiint_V \bar{h}_m^* \cdot \mu \cdot \bar{h}_m dV = \frac{1}{\nu_m^2}\iiint_V \text{curl}\,\bar{e}_m^* \cdot \mu^{-1}\cdot \text{curl}\,\bar{e}_m dV = \iiint_V \bar{e}_m^* \cdot \epsilon \cdot \bar{e}_m dV = \frac{1}{\nu_m^2}\iiint_V \text{curl}\,\bar{h}_m^* \cdot \epsilon^{-1}\cdot \text{curl}\,\bar{h}_m dV. \quad (18)$$

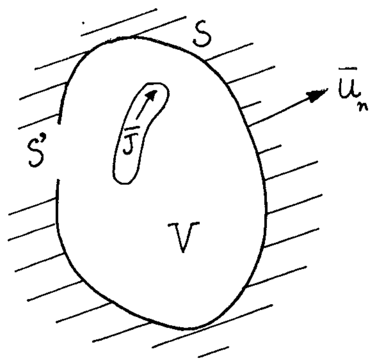

Fig. 1—Cavity filled with anisotropic medium.

3) That a complex square integrable vector admits the expansion

$$\bar{a} = \sum_m \bar{g}_m \frac{\iint_S \psi_m^* \bar{n}\cdot\mu\cdot\bar{a}\,dS - \iiint_V \psi_m^* \operatorname{div}(\mu\cdot\bar{a})\,dV}{\lambda_m^2 \iiint_V \psi_m^*\psi_m\,dV}$$

$$+ \sum_m \bar{h}_m \frac{\iiint_V \operatorname{curl}\bar{h}_m^*\cdot\epsilon^{-1}\cdot\operatorname{curl}\bar{a}\,dV}{\nu_m^2 \iiint_V \bar{h}_m^*\cdot\mu\cdot\bar{h}_m\,dV}. \quad (19)$$

Clearly, the $\bar{g}_m$ form a complete set for irrotational vectors, and the $\bar{h}_m$ for vectors $\bar{a}$ such that $\mu\cdot\bar{a}$ is tangent to the boundary and $\operatorname{div}(\mu\cdot\bar{a})=0$.

IV. Application to Cavity Fields

Consider a cavity (Fig. 1) excited through a hole S' (in which $\bar{n}\times\bar{E}$ is given) and by electric and magnetic currents $\bar{J}$ and $\bar{J}_m$. If we expand $\bar{E}$, $\epsilon^{-1}\cdot\bar{J}$ and $\epsilon^{-1}\cdot\operatorname{curl}\bar{H}$ in terms of electric eigenvectors, $\bar{H}$, $\mu^{-1}\cdot\bar{J}_m$ and $\mu^{-1}\cdot\operatorname{curl}\bar{E}$ in terms of magnetic eigenvectors, and insert the expansions in Maxwell's equations

$$\mu^{-1}\cdot\operatorname{curl}\bar{E} = -j\omega\mu_0\bar{H} - \mu^{-1}\cdot\bar{J}_m$$

$$\epsilon^{-1}\cdot\operatorname{curl}\bar{H} = j\omega\epsilon_0\bar{E} + \epsilon^{-1}\cdot\bar{J},$$

we find, after a few steps similar to those used for evacuated cavities,[1]

$$\bar{E} = -\frac{1}{j\omega\epsilon_0}\sum_m \bar{f}_m \frac{\iiint_V \bar{f}_m^*\cdot\bar{J}\,dV}{\iiint_* \bar{f}_m^*\cdot\epsilon\cdot\bar{f}_m\,dV}$$

$$+ \sum_m \frac{\bar{e}_m}{k^2-\nu_m^2}\left[\nu_m \frac{\iiint_V \bar{h}_m^*\cdot\bar{J}_m\,dV + \iint_S \bar{h}_m^*\cdot(\bar{n}\times\bar{E})\,dS}{\iiint_V \bar{h}_m^*\cdot\mu\cdot\bar{h}_m\,dV} + j\omega\mu_0 \frac{\iiint_V \bar{e}_m^*\cdot\bar{J}\,dV}{\iiint_V \bar{e}_m^*\cdot\epsilon\cdot\bar{e}_m\,dV}\right]$$

$$\bar{H} = -\frac{1}{j\omega\mu_0}\sum_m \bar{g}_m \frac{\iiint_V \bar{g}_m^*\cdot\bar{J}_m\,dV + \iint_S \bar{g}_m^*\cdot(\bar{u}_n\times\bar{E})\,dS}{\iiint_V \bar{g}_m^*\cdot\mu\cdot\bar{g}_m\,dV}$$

$$+ \sum_m \frac{\bar{h}_m}{k^2-\nu_m^2}\left[j\omega\epsilon_0 \frac{\iiint_V \bar{h}_m^*\cdot\bar{J}_m\,dV}{\iiint_V \bar{h}_m^*\cdot\mu\cdot\bar{h}_m\,dV} - \nu_m \frac{\iiint_V \bar{e}_m^*\cdot\bar{J}\,dV}{\iiint_V \bar{e}_m^*\cdot\epsilon\cdot\bar{e}_m\,dV} + j\omega\epsilon_0 \frac{\iint_S \bar{h}_m^*\cdot(\bar{n}\times\bar{E})\,dS}{\iiint_V \bar{h}_m^*\cdot\mu\cdot\bar{h}_m\,dV}\right]. \quad (20)$$

These formulas are quite similar to those obtained for evacuated cavities, and most of the comments made about the latter can be duplicated here, for example, that the electric currents should be parallel to the lines of the electric field to efficiently excite any given mode. We do not insist on this aspect, but notice that the formulas of (20) provide a purely formal solution of our problem. There remains the formidable task of actually determining the eigenvectors for any given geometry and disposition of gyrotropic material (in certain cases one might be satisfied with the structure of a single mode if operation at a resonant frequency is considered). This problem is not within the province of the present paper, and we shall only mention that cylindrical structures in the form of terminated waveguides are those for which the analysis can progress most satisfactorily.[9–12]

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An Impedance Transformation Method for Finding the Load Impedance of a Two-Port Network*

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Summary—**An unknown load impedance terminating a lossy two-port junction can be calculated if the input impedance and junction parameters are known. It is to be shown that there exists a linear relationship, dependent upon two calibration constants, between the input reflection coefficient and a modified reflection coefficient of the load. Applying the linear transformation to the junction input impedance permits evaluation of the unknown load impedance. Calibration is accomplished by terminating the transmission line in at least three different reactances and measuring the corresponding input reflection coefficients. These data plot into the usual circular configuration on a Smith chart from which the necessary calibration data is obtained. When several load reactances are used, the calibration accuracy can be considerably increased, since the averaging advantage of plotting a mean straight line is utilized. Furthermore, once the junction has been calibrated, its equivalent T-network impedances and scattering coefficients may be found.**

I. INTRODUCTION

AN UNKNOWN load impedance which terminates a two-port junction can be calculated if the input impedance and parameters of the junction are known. Several methods are currently available for determining the network parameters, such as the three-point method, canonical method, and the scattering-matrix method.[1–10]

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